Period 4, Dec 3, 2024

The hour hand of a clock suggests a **ray**, a part of a line that has only one endpoint and extends forever in the opposite direction. An **angle** is formed by two rays that have a common endpoint. One ray is called the **initial side** and the other the **terminal side**.

A rotating ray is often a useful way to think about angles. The ray in **Figure 4.1** rotates from 12 to 2. The ray pointing to 12 is the **initial side** and the ray pointing to 2 is the **terminal side**. The common endpoint of an angle's initial side and terminal side is the **vertex** of the angle.

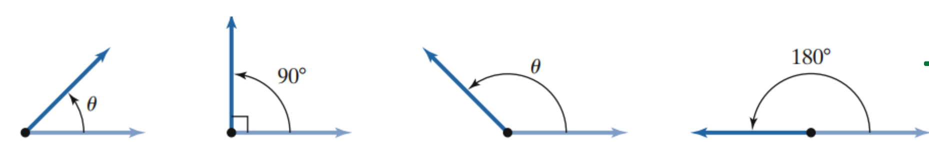
Standard Position of Angles

An angle is in **standard position** if

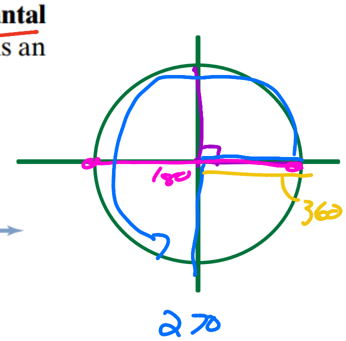
- its vertex is at the origin of a rectangular coordinate system and
- its initial side lies along the positive x-axis.

direction. **Positive angles** are generated by counterclockwise rotation. Thus, angle α is positive. By contrast, the arrow in **Figure 4.3(b)** shows that the rotation from the initial side to the terminal side is in the clockwise direction. **Negative angles** are generated by clockwise rotation. Thus, angle θ is negative.

has a terminal side that lies on the negative y-axis. An angle is called a **quadrantal angle** if its terminal side lies on the x-axis or on the y-axis. Angle β in **Figure 4.4** is an example of a quadrantal angle.



(a) Acute angle $(0° < \theta < 90°)$ **(b) Right angle** $(\frac{1}{4}$ rotation) **(c) Obtuse angle** $(90° < \theta < 180°)$ **(d) Straight angle** $(\frac{1}{2}$ rotation)



Definition of a Radian

One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.

The **radian measure** of any central angle is the length of the intercepted arc divided by the circle's radius. In **Figure 4.7(a)**, the length of the arc intercepted by angle β is double the radius, r. We find the measure of angle β in radians by dividing the length of the intercepted arc by the radius.

$$\beta = \frac{\text{length of the intercepted arc}}{\text{radius}} = \frac{2r}{r} = 2$$

In **Figure 4.7(b)**, the length of the intercepted arc is triple the radius, r. Let us find the measure of angle γ:

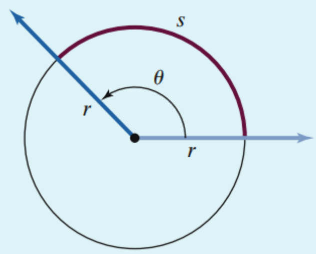
$$\gamma = \frac{\text{length of the intercepted arc}}{\text{radius}} = \frac{3r}{r} = 3.$$

Thus, angle γ measures 3 radians.

Radian Measure

Consider an arc of length s on a circle of radius r. The measure of the central angle, θ, that intercepts the arc is

$$\theta = \frac{s}{r} \text{ radians.}$$



Conversion between Degrees and Radians

Using the basic relationship π radians $= 180°$,

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi \text{ radians}}{180°}$.
2. To convert radians to degrees, multiply radians by $\dfrac{180°}{\pi \text{ radians}}$.

Convert the angle in degrees to radians. Round to two decimal places.

17°

degree Radians

$$\frac{180}{17} = \frac{\pi}{X}$$

$$\frac{17\pi}{180} = \frac{180X}{180}$$

$$0.296 = \frac{53.41}{180} = \frac{17\pi}{180} = X$$

0.30

degree Rad

Convert the angle in radians to degrees.

$\frac{8\pi}{5}$ radians

$$\frac{180}{x} = \frac{\pi}{\frac{8\pi}{5}}$$

$$\pi x = 180\left(\frac{8\pi}{5}\right)$$

$$\pi x = 36 \cdot \cancel{5} \cdot \frac{8\pi}{\cancel{5}}$$

$$\frac{\cancel{\pi} x}{\pi} = \frac{36 \cdot 8 \cdot \cancel{\pi}}{\pi}$$

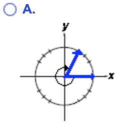
$$x = 288^\circ$$

Draw the angle in standard position. State the quadrant in which the angle lies. Work the exercise without converting to degrees.

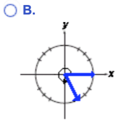
$\frac{5\pi}{6}$

Choose the correct graph below.

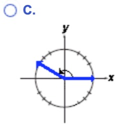
A.



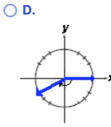
B.



C.



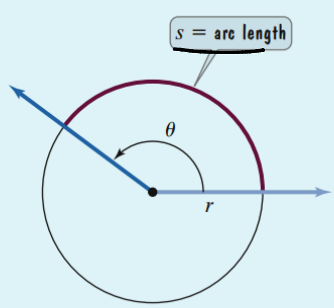
D.



The Length of a Circular Arc

Let r be the radius of a circle and θ the nonnegative radian measure of a central angle of the circle. The length of the arc intercepted by the central angle is

$$s = r\theta.$$



A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 120°.

Definitions of Linear and Angular Speed

If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its **linear speed** is

$$v = \frac{s}{t}, = \frac{\text{arc length}}{\text{Time}} = \frac{\text{Radians} \cdot \text{Radius}}{\text{Time}}$$

where s is the arc length given by $s = r\theta$, and its **angular speed** is

$$\omega = \frac{\theta}{t}. = \frac{\text{angle}}{\text{Time}}$$

Linear Speed in Terms of Angular Speed

The linear speed, v, of a point a distance r from the center of rotation is given by

$$v = r\omega,$$

where ω is the angular speed in radians per unit of time.

In words: Linear speed is the radius times the angular speed.

degrees

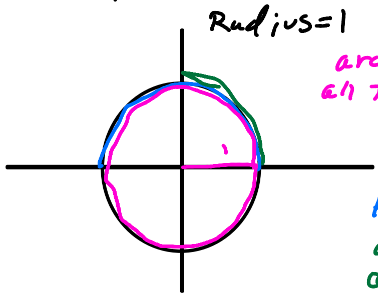
Radians

$360 = 2\pi$

$180 = \pi$

$90 = \frac{\pi}{2}$

Unit Circle



arc length $= 2\pi$ all the way around

arc length $= \pi$ Half way around

arc length of Quarter around $= \frac{\pi}{2}$

$\text{Cir} = 2\pi r = 2\pi \cdot 1 = 2\pi$

deg Rad

$30^\circ = \frac{\pi}{6}$

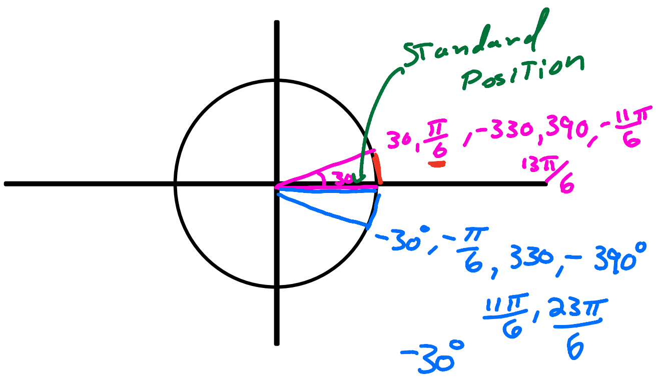
$120 = \frac{2\pi}{3}$

$135 = \frac{3\pi}{4}$

$270 = \frac{3\pi}{2}$

$210 = \frac{7\pi}{6}$

$\frac{\pi}{x} = \frac{180}{30}$



around 1 Time

$-\frac{\pi}{6} + 2\pi = -\frac{\pi}{6} + \frac{12\pi}{6} = \frac{11\pi}{6}$

around Twice

$-\frac{\pi}{6} + 4\pi = -\frac{\pi}{6} + \frac{24\pi}{6} = \frac{23\pi}{6}$

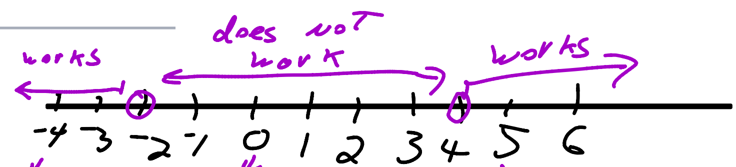
Find the domain of the logarithmic function.

$$f(x) = \log\left(\frac{x+2}{x-4}\right)$$

No negatives
No zero

$x \neq 4, -2$

$(-\infty, -2) \cup (4, \infty)$



Test
$\frac{-4+2}{-4-4}$
$\frac{-2}{-8}$
$\frac{1}{4}$

Test
$\frac{0+2}{0-4}$
$\frac{2}{-4}$
$-\frac{1}{2}$

Test
$\frac{5+2}{5-4}$
$\frac{7}{1}$
7

Without using a calculator, find the exact value of the following expression.

$$\frac{\overset{2}{\log_4 16} - \overset{0}{\log_\pi 1}}{\underset{2}{\log_{5\sqrt{2}} 50} - \underset{(+3)}{\log 0.001}} = \frac{2}{5}$$

$\log_4 16 = a$

$4^a = 16$

$a = 2$

$\log_\pi 1 = b$

$\pi^b = 1$

$b = 0$

$\log_{5\sqrt{2}} 50 = c$

$5\sqrt{2}^{\,c} = 50$

$(\sqrt{2 \cdot 5 \cdot 5})^c = 50$

$(\sqrt{50})^c = 50$

$c = 2$

$\log_{10} 0.001 = d$

$10^d = 0.001 = 10^{-3}$

$d = -3$

Use transformations of the graph of f(x) = **log** x to graph the given function. Graph and give the equation of the asymptote. Use the graphs to determine the function's domain and range.

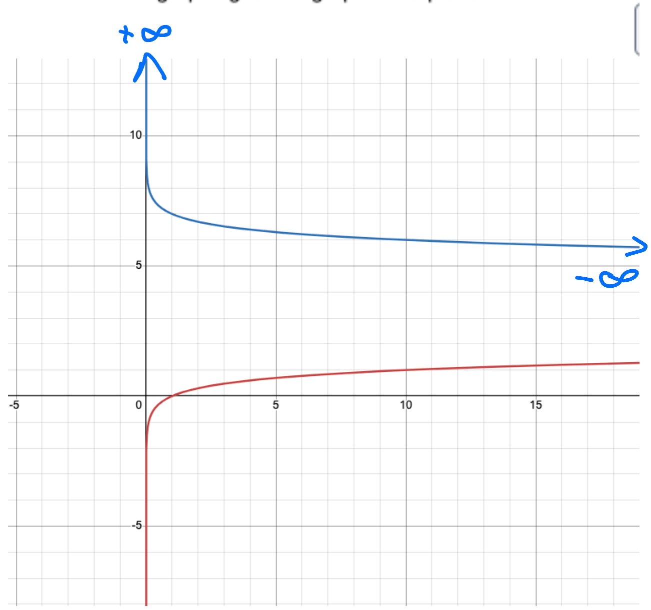
g(x) = 7 − **log** x Flips over X-axis

up 7

Determine the transformations that are needed to go from f(x) = **log** x to the given graph. Choose the correct answer below.

- A. The graph of f(x) = **log** x should be reflected about the x-axis and horizontally shifted 7 units to the right.
- B. The graph of f(x) = **log** x should be reflected about the y-axis and horizontally shifted 7 units to the left.
- C. The graph of f(x) = **log** x should be reflected about the y-axis and vertically shifted 7 units downward.
- D. The graph of f(x) = **log** x should be reflected about the x-axis and vertically shifted 7 units upward.

Graph g(x) = 7 − **log** x. Graph the asymptote of g(x) as a dashed line. Use the graphing tool to graph the equations.



What is the vertical asymptote of g(x)?

x = 0

(Type an equation.)

What is the domain of g(x) = 7 − **log** x?

(0,∞)

(Simplify your answer. Type your answer in interval notation.)

What is the range of g(x) = 7 − **log** x?

(−∞,∞)

(Simplify your answer. Type your answer in interval notation.)

$x = y^2 - 1$
$+1 \qquad +1$

$\sqrt{x+1} = \sqrt{y^2}$

$\pm\sqrt{x+1} = y$

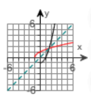
Given the function $f(x) = x^2 - 1,\ x \ge 0$, complete parts a through c.

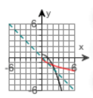
(a) Find an equation for $f^{-1}(x)$.
(b) Graph f and f^{-1} in the same rectangular coordinate system.
(c) Use interval notation to give the domain and the range of f and f^{-1}.

(a) Find $f^{-1}(x)$.

$f^{-1}(x) = \sqrt{x+1}$
(Type an exact answer, using radicals as needed.)

(b) Graph f and f^{-1} in the same coordinate system. Choose the correct graph below.

A.


B.


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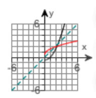

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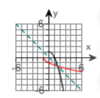
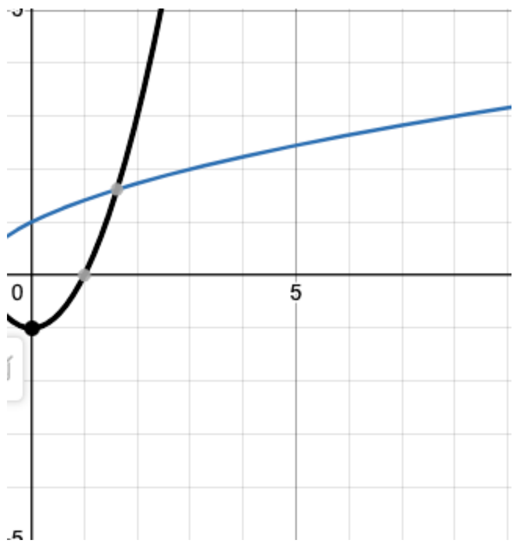
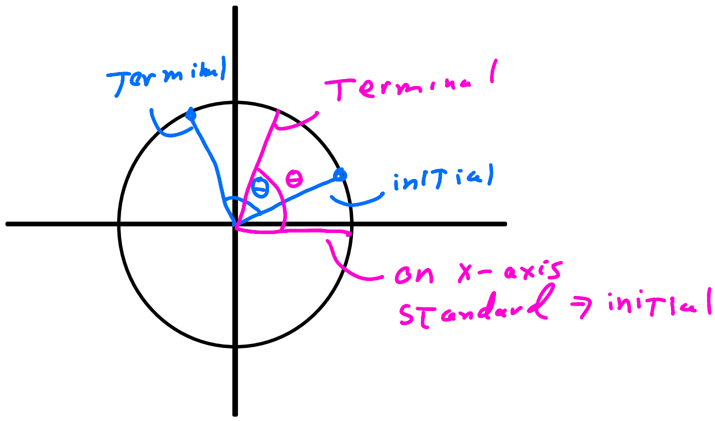


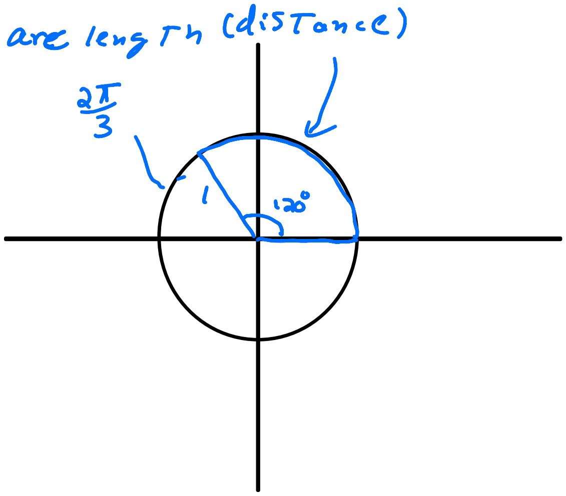
(c) State the domain and range of f and f^{-1} using interval notation.

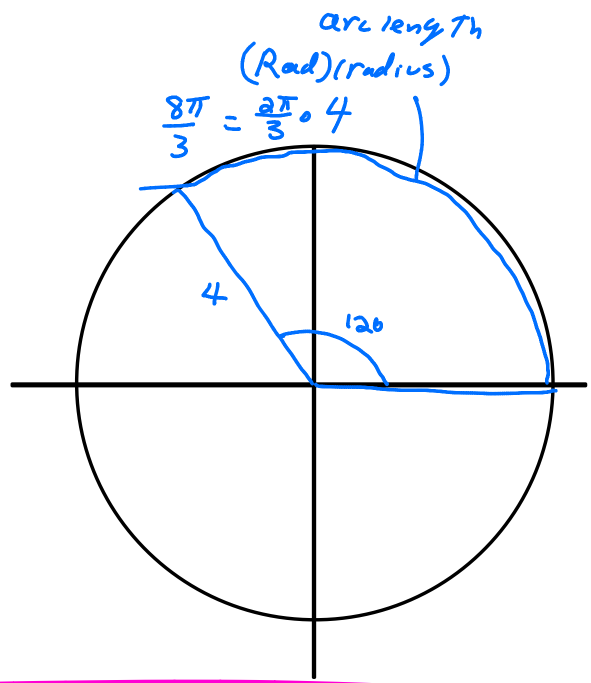
The domain of f(x) is $[0,\infty)$, and the range of f(x) is $[-1,\infty)$.

The domain of $f^{-1}(x)$ is $[-1,\infty)$, and the range of $f^{-1}(x)$ is $[0,\infty)$.









arc length of central angle = 63°
with Radius = 6

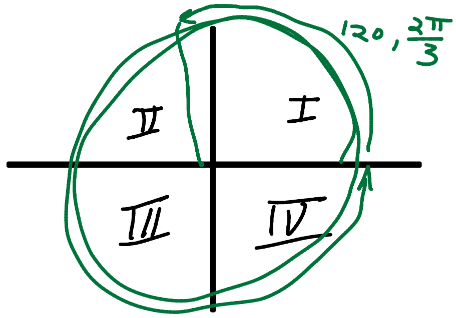
1st step degree to Radians

$$\frac{180}{63} = \frac{\pi}{x} \Rightarrow \frac{63\pi}{180} = \frac{\cancel{180}x}{\cancel{180}} \Rightarrow \frac{63\pi}{180} = x = \frac{7\pi}{20}$$

2nd step Radians • Radius

$$\frac{63\pi}{180} \cdot 6 = \frac{6 \cdot 7 \cdot \cancel{9} \cdot \pi}{\cancel{9} \cdot 20} = \frac{3 \cdot \cancel{2} \cdot 7\pi}{\cancel{2} \cdot 10} = \frac{21\pi}{10}$$

Quad



$\frac{14\pi}{3}$ which Quad II

$4\frac{2}{3}\pi$

around Twice

distance = Rate · Time

speed

$\frac{\text{distance}}{\text{Time}} = \text{Rate}$

if I walk 10 steps in 3sec, what is my speed?

30 feet

$\text{Rate} = 10\ \text{FT}/\text{sec} = \frac{30\ \text{FT}}{3\ \text{sec}}$

Linear speed Car Tires radius = 2FT

angular speed 7 Rev Per sec

$7 \cdot 2\pi = 14\pi$

$14\pi \cdot 2 = 28\pi\ \text{FT}/\text{sec}$